460-2 International Economics

Lecture notes 2: Nominal rigidities

1 A simple model with nominal rigidites

We now add nominal rigidities to the picture, so we can discuss how financial flows can affect real activity in the short run.

Consider a small country, populated by infinitely lived consumers with utility function

$$\sum_{t=0}^{\infty} \beta^t \left(\log C_t - \frac{\psi}{1+\phi} N_t^{1+\phi} \right).$$

Consumption C_t is a Cobb-Douglas aggregate of a home good C_{ht} and a foreign good C_{ft}

$$C_t = \xi C_{ht}^{\omega} C_{ft}^{1-\omega}.$$

Home good is the aggregate of a continuum of differentiated goods

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

and its price index is

$$P_{ht} = \left(\int_0^1 P_{ht}(j)^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}}.$$

Each firm j uses the linear technology

$$Y_t(j) = A_t N_t(j).$$

The foreign good is in perfectly elastic supply at the price P_{ft}^* in foreign currency. The nominal exchange rate is $\mathcal{E}_t$, so the price of the foreign good in domestic currency is $\mathcal{E}_t P_{ft}^*$.

We will consider an economy with no uncertainty, except possibly an unexpected shock at date t.

Since the representative consumer receives the entire domestic income $P_{ht}Y_t$ in the form of wages or profits, his budget constraint is

$$Q_t B_{t+1} + P_t C_t - P_{ht} Y_t = B_t,$$

where B_t is the country's asset position in domestic currency and $Q_t = 1/(1+i_t)$ is the price of domestic currency bonds (i_t is the domestic nominal interest rate). We don't need to account explicitly for bonds denominated in foreign currency because consumers will be perfectly indifferent between the two bonds, by the arbitrage condition (4) below.

Consumer optimality implies that the domestic demand for the home good is

$$C_{ht} = \omega \left(\frac{P_{ht}}{P_t} \right)^{-1} C_t, \tag{1}$$

where the domestic consumer price index P_t includes the price of the home and foreign good and is

$$P_t = P_{ht}^{\omega} \left(\mathcal{E}_t P_{ft}^* \right)^{1-\omega}. \tag{2}$$

The demand of home goods by the rest of the world is given by

$$C_{ht}^* = (1 - \omega) \left(\frac{P_{ht}}{\mathcal{E}_t P_t^*} \right)^{-1} C_t^*. \tag{3}$$

This captures the idea that preferences are symmetric across countries and each country in the rest of the world spends a fraction $(1 - \omega)$ in goods from other countries. We are in a small open economy, so consumption of home goods is a negligible fraction of spending in the rest of the world and the world CPI P_t^* is unaffected by P_{ht}. This specification is consistent with an explicit model with a continuum of countries as in Gali and Monacelli.

Combining (1) and (3) we can write the equilibrium condition in the market for the home good as

$$Y_t = \omega \left(\frac{P_{ht}}{P_t} \right)^{-1} C_t + (1 - \omega) \left(\frac{P_{ht}}{\mathcal{E}_t P_t^*} \right)^{-1} C_t^*.$$

The foreign interest rate is i_t^*. There is perfect capital mobility and no uncertainty so the uncovered interest parity condition holds exactly and is

$$1 + i_t = \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} (1 + i_t^*). \tag{4}$$

We introduce nominal rigidities by assuming that firms set prices one period in advance. Wages are flexible.

Optimality for consumers and firms is captured by three equations. Consumer optimality implies the consumer Euler equation:

$$C_t^{-1} = \beta (1 + i_t) \frac{P_t}{P_{t+1}} C_{t+1}^{-1}. \tag{5}$$

The optimal labor supply condition:

$$\frac{W_t}{P_t} C^{-\sigma} = \psi N_t^{\phi}. \tag{6}$$

Optimality for price setters implies:

$$P_{ht}(j) = \frac{\varepsilon}{\varepsilon - 1} \frac{W_t}{A_t}, \tag{7}$$

for all producers that can adjust their price at date t.

For the rest of the analysis, it is useful to define the relative price of the home good in terms of foreign goods

$$p_t \equiv \frac{P_{ht}}{\mathcal{E}_t P_{ft}^*},$$

and to express the relative prices of the home and foreign good in terms of domestic consumption as

$$P_{ht}/P_t = p_t^{1-\omega},$$
$$P_{ft}^*\mathcal{E}_t/P_t = p_t^{-\omega}.$$

1.1 Flexible prices

We consider first the case of fully flexible prices.

Assume the initial asset position is given in foreign currency, so

$$B_0 = \mathcal{E}_0 \hat{B}_0,$$

for a fixed value of $\hat{B}_0$.

Equilibrium is characterized as follows. The intertemporal budget constraint can be written as

$$\sum Q_{t|0}^* \left(\frac{P_t C_t - P_{ht} Y_t}{\mathcal{E}_t} \right) = \hat{B}_0$$

where $Q_{t|0}^*$ is the date 0 price of a foreign bond that pays at date t. We can rewrite it as

$$\sum Q_{t|0}^* \frac{P_{ft}}{P_{f0}} \left(p_t^\omega C_t - p_t Y_t \right) = 0.$$

The Euler equation combined with UIP can be written as

$$C_t^{-1} = \beta \left(1 + i_t^* \right) \frac{P_t/\mathcal{E}_t}{P_{t+1}/\mathcal{E}_{t+1}} C_{t+1}^{-1},$$

or

$$p_{t+1}^\omega C_{t+1} = \beta \left(1 + i_t^* \right) \frac{P_{ft}}{P_{ft+1}} p_t^\omega C_t.$$

The labor supply and price setting equations can be combined to get

$$\frac{\varepsilon - 1}{\varepsilon} \frac{P_{ht}}{P_t} C_t^{-1} = \psi N_t^\phi,$$

or

$$\frac{\varepsilon - 1}{\varepsilon} A_t p_t^{1-\omega} C_t^{-1} = \psi N_t^\phi. \tag{8}$$

The good market equilibrium can be written as

$$Y_t = \omega p_t^{\omega-1} C_t + (1 - \omega) p_t^{-1} \frac{P_t^*}{P_{ft}} C_t^*. \tag{9}$$

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Given C_t we can find values for p_t, N_t, Y_t solving the nonlinear equations (8)-(9). So given a conjectured sequence $\{C_t\}$ we get a sequence $\{p_t\}$ and we can check if the Euler equation and intertemporal budget constraint are satisfied. We will proceed under the assumption that there is a unique equilibrium sequence $\{C_t, Y_t, N_t, p_t\}$.

The real side of the economy is pinned down with no reference to the nominal side and monetary policy.

1.2   Pre-set prices

Consider now the case in which prices are set one period in advance. We consider the case of an unexpected shock to monetary policy at date T. From $T+1$ onward there is perfect foresight, so equation

$$P_{ht} = \frac{\varepsilon}{\varepsilon - 1} W_t,$$

holds for $t = T+1, T+2, \dots$ but does not need to hold at date T. This means that, given $\hat{B}_{T+1}$, the allocation from $T+1$ onward is given.

We now make two assumptions that simplify the analysis:

- the country enters period T with a zero net foreign asset position $\hat{B}_T = 0$;

- the real interest rate in terms of foreign goods is constant and equal to $1/\beta$ in all periods
$$\beta \left(1 + i_t^*\right) \frac{P_{ft}}{P_{ft+1}} = 1,$$

- foreign spending in terms of foreign goods is constant
$$\frac{P_t^* C_t^*}{P_{ft}} = const.$$

Given these assumptions we can make the conjecture that $\hat{B}_t = 0$ for all $t \geq T$ irrespective of monetary policy at date T. Under this conjecture, the real allocation from $T+1$ onwards is independent of what happens at date T.

The equilibrium at date T is then characterized by the following three equations:

$$p_{T+1}^{\omega} C_{T+1} = \beta \left(1 + i_T^*\right) \frac{P_{fT}}{P_{fT+1}} p_T^{\omega} C_T,$$

$$Y_T = \omega p_T^{\omega-1} C_T + (1 - \omega) p_T^{-1} \frac{P_T^*}{P_{fT}} C_T^*$$

$$Y_T = A_T N_T$$

where p_{T+1} and C_{T+1} are given by the flexible price allocation. These three equations define a one-dimensional set of feasible values for $\{C_T, Y_T, N_T, p_T\}$.

Choosing p_t, we get the values of the other 3 variables. This means that monetary policy now has one degree of freedom. We can describe monetary policy in terms of the real exchange rate it achieves today p_T, or go a step forward and describe it in terms of the nominal interest rate needed to achieve that exchange rate, as we shall do shortly. This shows that exchange rate policy and monetary policy are tightly linked, if the only instrument of the policy maker is a traditional monetary policy instrument like the nominal rate (and not, for example, capital controls or other tools). In this model there is no independent room for currency interventions.

Let's check that the current account is zero irrespective of monetary policy. Using the goods market equilibrium we have

$$p_t Y_t - p_t^\omega C_t = (1 - \omega) \left(p_t^\omega C_t - \frac{P_t^*}{P_{ft}} C_t^* \right).$$

Moreover, the Euler equation implies that $p_t^\omega C_t$ is constant over time. Using the intertemporal budget constraint we then need

$$p_t Y_t = p_t^\omega C_t = \frac{P_t^*}{P_{ft}} C_t^*$$

at all t.

Let us now go back to how monetary policy is implemented. Writing the Euler equation as

$$C_T^{-1} = \beta (1 + i_T) \frac{P_T}{P_{T+1}} C_{T+1}^{-1},$$

we see that given a target for the exchange rate and a corresponding target for consumption C_T, this equation tells us the real interest rate needed to achieve that target (recall that C_{T+1} is given). Therefore monetary policy can implement the desired allocation by choosing

$$(1 + i_T) \frac{P_T}{P_{T+1}}.$$

In this simple economy, the desired real interest rate can be achieved either by changing the nominal rate or by changing target inflation between T and $T+1$. With more general specifications for price stickiness, that's not the case. Here we focus on the interpretation in which the inflation target is kept at 0 and monetary policy intervenes by changing the nominal rate. The effect of increasing the nominal rate is to reduce consumer spending. Through the UIP an increase in the nominal rate also leads to a nominal and a real appreciation since we have

$$1 + i_T = \frac{\mathcal{E}_{T+1}/P_{T+1}}{\mathcal{E}_T/P_T} \frac{P_{T+1}}{P_T} (1 + i_T^*),$$

and $\mathcal{E}_{T+1}/P_{T+1}$ and P_T are given. Therefore, on the goods market output decreases for two reasons: an increase in domestic spending and an increase in exports, driven by the depreciation.

Summing up: an increase in the home nominal rate increases domestic spending, appreciates the exchange rate, contracts output and has no effects on the current account. The zero effect on the current account arises because the reduction in imports driven by the contraction in domestic spending is exactly matched by a reduction in exports due to the real exchange rate appreciation.

1.3 Welfare

Since the future allocations are not affected by monetary policy we can focus on welfare at T which is given by

$$\log C - \frac{\psi}{1+\phi} N^{1+\phi}$$

where

$$C = p^{-\omega} C^*, Y = C^*/p, N = Y/A$$

Substituting we want to maximize

$$\omega \log N - \frac{\psi}{1+\phi} \left(N\right)^{1+\phi}$$

so the FOC is

$$\frac{\omega}{N} = \psi N^{\phi}$$

There are two differences with the flexible price allocation in which

$$\frac{\varepsilon - 1}{\varepsilon} \frac{1}{N} = \psi N^{\phi}$$

There is no monopolistic distortion, but there is a distortion due to the ability to manipulate our domestic terms of trade. Monetary policy wants to induce more production to counteract the monopolistic distortion, but it wants to induce less production to increase terms of trade in our favor. If we introduce a subsidy/tax to producers equal to

$$\frac{\varepsilon}{\varepsilon - 1} \omega - 1,$$

we control both distortions in steady state and monetary policy can focus on replicating the flexible price allocation.

1.4 Open problem

Consider the more general case in which the central bank has two tools:

- a capital control,
- monetary policy.

Argue that then the welfare problem takes the form of maximizing

$$\log C - \frac{\psi}{1+\phi} N^{1+\phi} + \beta V\left(\hat{B}\right)$$

and that the planner has two degrees of freedom. Characterize optimal policy. Discuss whether optimal capital controls are zero and monetary policy replicates flexible prices. See how you answers change if you relax the assumption

$$\beta\left(1+i_t^*\right) \frac{P_{ft}}{P_{ft+1}} = 1.$$

You can assume that the subsidy to producers is constant over time and set to be optimal in steady state and ask how monetary policy and capital controls optimally respond to a temporary shock to i_T^*.

1.5 Cole and Obstfeld

- A small country with endowment process $\{Y_{ht}\}$ of home good.

- World demand for home good is

$$(1-\omega) P_{ht}^{-1} C^*$$

- Intertemporal preferences

$$\sum \beta^t \log(C_{ht}^\omega C_{ft}^{1-\omega})$$

- Intertemporal trade

$$\sum q^t (P_{ht}(C_{ht} - Y_{ht}) + C_{ft}) = B_0$$

$$q^t \frac{\omega}{C_{ht}} = \lambda \beta^t P_{ht}$$

$$q^t \frac{1-\omega}{C_{ft}} = \lambda \beta^t$$

- Assume

$$\beta = q$$

- The first foc requires constant spending in home good $P_{ht} C_{ht}$

- The second foc requires constant spending on foreign good C_{ft}

- So total spending is constant, denote it by X

- Then demand for home good is

$$\omega X P_{ht}^{-1} + (1-\omega) P_{ht}^{-1} C^* = Y_{ht}$$

- This implies that the equilibrium price is inversely proportional to Y_{ht} and income is constant and equal to

$$P_{ht}Y_{ht} = \omega X + (1 - \omega) C^*$$

- Intuition: when the home good's supply increases the price drops proportionally, so domestic income in term of foreign goods remains constant and there is no scope for insurance

- To complete the analysis, from the budget constraint we have

$$X = P_h Y_h + (1 - q) B_0$$

which implies

$$X = C^* + \frac{1 - q}{1 - \omega} B_0$$

2 Overvaluation and multiple equilibria

- Suppose the central bank is committed to a fixed exchange rate

$$\mathcal{E}_t = \bar{\mathcal{E}}$$

- We want to study how this committment can come under attack, if inflation expectations are out of line

- Consider a version of the model with two group of firms

- A mass α cannot change price, price is pre-set at $\bar{P}_h$

- A mass $1 - \alpha$ (flex price firms) can change price at date 0

- Game at date 0:

    - Flex price firms set price $\hat{P}_{h0}$ forming expectations about C_0 and N_0
    - Central bank sets i_0 and $\mathcal{E}_0$ and quantities are determined

- When setting $\hat{P}_{h0}$ firms are also forming expectations about other firms' prices

2.1 Equilibrium

- Backward induction, given $\hat{P}_{h0}$ solve the central bank problem

- Price of home good is

$$P_{h0} = \left(\alpha \bar{P}_h^{1-\varepsilon} + (1 - \alpha) \hat{P}_{h0}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$$

- Given total demand Y_0 for home goods the demand for the goods produced by fix and flex firms are

$$\left(\frac{\bar{P}_h}{P_{h0}}\right)^{-\varepsilon} Y_0 \text{ and } \left(\frac{\hat{P}_{h0}}{P_{h0}}\right)^{-\varepsilon} Y_0$$

- So aggregating and using linearity of the technology we have that total labor demand is

$$N_0 = J_0 Y_0$$

where

$$J_0 \equiv \alpha \left(\frac{\bar{P}_h}{P_{h0}}\right)^{-\varepsilon} + (1-\alpha) \left(\frac{\hat{P}_{h0}}{P_{h0}}\right)^{-\varepsilon}$$

- By choosing the nominal interest the central bank can choose any triple C_0, p_0 and Y_0 that satisfies

$$C_0 = p_0^{-\omega} \tag{10}$$
$$Y_0 = p_0^{-1} \tag{11}$$

exactly as in Section 1

- Moreover the value of B_1 and the continuation welfare are independent of central bank policy so we can focus on welfare at date 0

$$U_0 = \log C_0 - \frac{\psi}{1+\phi} N_0^{1+\phi}$$

- Expressing it in terms of Y_0 we have

$$\omega \log Y_0 - \frac{\psi}{1+\phi} (J_0 Y_0)^{1+\phi}$$

- If the central bank decides to float, its optimality condition is

$$\frac{\omega}{Y_0} = \psi J_0^{1+\phi} Y_0^{\phi}$$

- That is, the central bank best response is

$$Y_0 = (\omega/\psi)^{\frac{1}{1+\phi}} J_0^{-1}$$

- If central bank sticks to peg then

$$p_0 = \frac{P_{h0}}{\bar{\mathcal{E}}}$$

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- Gain from floating

$$\Delta W(\hat{P}_h) = \max_Y \left\{ \omega \log Y - \frac{\psi}{1+\phi} \left(\mathcal{J}(\hat{P}_h) Y \right)^{1+\phi} \right\} -$$
$$\left[\omega \log \mathcal{Y}(\hat{P}_h) - \frac{\psi}{1+\phi} \left(\mathcal{J}(\hat{P}_h) \mathcal{Y}(\hat{P}_h) \right)^{1+\phi} \right]$$

- Go backward to price setters optimality

- Price setters choose prices in anticipation of $C_0, N_0, \mathcal{E}_0$

- Optimality of price setters, together with equilibrium wages

$$\hat{P}_{h0} = P_0 C_0 N_0^\phi$$

where
$$P_0 = P_{h0}^\omega \mathcal{E}_0^{1-\omega}$$

- Assume
$$\omega = \psi$$

so if $\hat{P}_{h0} = \bar{P}_h = P_{h0}$ it is optimal for the central bank to implement the flexible price allocation
$$Y_0 = C_0 = p_0 = 1$$

- Assume
$$\bar{P}_h / \bar{\mathcal{E}} > 1$$

so currency is initially overvalued

2.2 Multiple equilibria

- Conjecture: equilibrium with

$$\hat{P}_{h0} = \bar{P}_h = P_{h0}$$

- Then $J_0 = 1$ and gain from floating is

$$\Delta W_{float} = \omega \log 1 - \frac{\psi}{1+\phi} -$$
$$\left[\omega \log \frac{\bar{\mathcal{E}}}{\bar{P}_h} - \frac{\psi}{1+\phi} \left(\frac{\bar{\mathcal{E}}}{\bar{P}_h} \right)^{1+\phi} \right]$$

- Price setters optimality holds because they expect $C_0 = N_0 = 1$ and $\mathcal{E}_0 = P_{h0} = \bar{P}_h$

$$\hat{P}_{h0} = P_0 C_0 N_0^\phi$$

where
$$P_0 = P_{h0}^\omega \mathcal{E}_0^{1-\omega} = \bar{P}_h$$

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- Suppose cost of floating is κ and satisfies

$$\kappa < \Delta W_{float}$$

then we have an equilibrium

- Can we have also an equilibrium with fixed exchange rates?

- Now price setters anticipate

$$C_0 = \left(\frac{\bar{\mathcal{E}}}{P_{h0}} \right)^{\omega}$$

$$Y_0 = \frac{\bar{\mathcal{E}}}{P_{h0}}$$

and

$$J_0 = \left[\alpha \bar{P}_{h0}^{-\varepsilon} + (1 - \alpha) \hat{P}_{h0}^{-\varepsilon} \right] P_{h0}^{\varepsilon}$$

and

$$P_0 = P_{h0}^{\omega} \bar{\mathcal{E}}^{1-\omega}$$

- So we have

$$\hat{P}_{h0} = P_0 C_0 N_0^{\phi} = P_{h0}^{\omega} \bar{\mathcal{E}}^{1-\omega} \left(\frac{\bar{\mathcal{E}}}{P_{h0}} \right)^{\omega} \left(\left[\alpha \bar{P}_{h0}^{-\varepsilon} + (1 - \alpha) \hat{P}_{h0}^{-\varepsilon} \right] P_{h0}^{\varepsilon} \frac{\bar{\mathcal{E}}}{P_{h0}} \right)^{\phi} =$$

$$= \bar{\mathcal{E}}^{1+\phi} \left(\frac{\alpha \bar{P}_h^{-\varepsilon} + (1 - \alpha) \hat{P}_{h0}^{-\varepsilon}}{\alpha \bar{P}_h^{1-\varepsilon} + (1 - \alpha) \hat{P}_{h0}^{1-\varepsilon}} \right)^{\phi}$$

- Graphically we can see this has unique fixed point and

$$\hat{P}_{h0} < \bar{\mathcal{E}} < \bar{P}_h$$

which implies

$$\frac{P_{h0}}{\bar{\mathcal{E}}} < \frac{\bar{P}_{h0}}{\bar{\mathcal{E}}}$$

- So output if fixed expected and fixed is realized is higher than output if float is expected and fixed is realized

- If fixed is expected there is some internal devaluation that helps

- This suggests that ΔW_{fix} will be lower than ΔW_{float}

- There are added complications in proving this inequality, due to the presence of J

- But numerically I always got $\Delta W_{fix} < \Delta W_{float}$

- Moreover the distance between the two depends on the initial degree of overvaluation, if $\frac{\bar{P}_{h0}}{\bar{\mathcal{E}}} = 1$ then $\Delta W_{fix} = \Delta W_{float} = 0$

- So it's possible to find a κ such that

$$\Delta W_{fix} < \kappa < \Delta W_{float}$$

so we have two equilibria